Introduction to Microeconomics

Pedro Hemsley

PPGE-IE-UFRJ

Based on Alexander Wolitzky lecture slides, available at MIT Opencourseware

Individual Decision-Making

- Economics studies interaction of individual decision-makers.
  - We begin with the theory of individual choice, and then proceed to study of how individuals interact in markets and other settings.
  - This setup is very general and is used in other fields: political science, sociology, social psychology.
  - Before turning to some particular economic models, let’s consider some basic assumptions we usually make in the theory of choice.

Individual Decision-Making

1. Choices are possible.
    - Seems obvious, but sets us apart from many models from 19th century sociology.
    - Important link to theory of agency in biology.
2. The decision maker is the individual.
    - (Usually) not a set of individuals such as family nor parts of an individual such as Id and Ego – but there has much development in this direction.
    - This is not the selfish agent assumption, which we can also make in particular settings.

Individual Decision-Making

3. Choices are made according to some criteria.
    - They are not purely random (at least not necessarily)
    - This is the same as saying as there is some objective. Could be anything: profit, utility, leisure, stability, time with loved ones, watching as many world cup matches as possible…
4. Choices are subject to constraints.
    - One cannot simply choose ‘everything’. At the very least there is the opportunity cost of time.
    - This is not necessarily a budget restriction: could be time, attention, memory, information, ability to process information, or external factors such as physical environment or laws, or simply other agents.
    - In this course, we will focus on a budget constraint, but this is just a particular case, and the absent restrictions are as important as the one(s) we will consider!

Individual Decision-Making

5. Consistency requirement: equilibrium

- Everyone is choosing according to the four previous assumptions at any point in time.
- In other words: given the criteria and the restrictions at some point in time, no one would like to change their decision at that moment.
- This is NOT about absence of movement or change. There could be mistakes and regrets.

Some difficult words

- This setup is an approach based on **optimization** and **equilibrium**.
- Sometimes we talk about **rational agents**.
- All these words have multiple meanings and lead to confusion.
- Optimization simply means that individuals make choices according to some criteria, given the relevant restrictions.
- But sometimes used as ‘perfect optimization’ or ‘hyper rationality’, which needs not be the case.

Some difficult words

•Rationality has different uses within microeconomics. First: similar to optimization under restrictions. Second: a particular set of basic assumptions on the decision-maker. We will use the latter.

•Sometimes (very often!) used in the sense of 'super computational power'. This is usually the case in basic microeconomics: we only have a budget constraint, meaning there are no cognitive constraints.

•This is often referred to as 'homo economicus' of 'homo rationalis' and is simply our lab rat: we study it not because it's realistic, but because it's much easier to study, and many things we learn carry over to 'actual' humans.

Some difficult words

- Equilibrium means there is no unilateral incentive to change in a given context.
- Sometimes used, even within economics, in the sense of physics: lack of movement, or some very stable movement. This is NOT the meaning of the word in microeconomics.

Mathematics, and Science in general

- Our assumptions generate a setup that may be analyzed mathematically. Our optimization problem will be written as something similar to:
- $Max_x\ f(x)$ subject to some restriction $g(x) \leq 0$.
- Yet, it is important to notice the difference between <u>language</u> and <u>tools</u>.
- We use mathematical language as in the example above. This does not mean we're using any mathematical tools yet: this could be written in plain English.
- Often we will use actual mathematical tools: if f and g are differentiable functions, then we may use first-order conditions.

Mathematics, and Science in general

•We'll make many other assumptions along the way. Pay attention to them: assumptions must be clearly understood.

•Assumptions have a tradeoff. On one hand, they take away generality: if we assume economic agents have perfect memory, we must be cautious when applying our model to agents without perfect memory.

•On the other hand, they allow us to better understand a (more restricted) setting.

•This is the tradeoff of the lab rat.

•In the end of the day, we use approximations, and we want predictions that can be tested to see whether these approximations are good enough.

Mathematics, and Science in general

- Keep in mind that external validity is always an issue, even when we have good empirical results, and this depends on how restrictive our assumptions are.

- Lastly, a point about representation: a model of reality is different from reality.

- Implication: solving a model in decision theory is different from actually making that decision. (Same reasoning holds for anything else in science.)

- Think of catching an object thrown to you (no actual functional analysis problem) or buying stuff at the grocery store (no actual lagrangian).

- This goes back to theory of agency in biology: consciousness, representation and abstraction.

Mathematics, and Science in general

- Important: if the modeler knows more than the agents in the model, this must be modeled too! It’s some form of restriction.

- Let’s turn to our optimization problem now. We will start with the utility maximization problem.

Utility Maximization

- Basic model of individual choice:
  - A decision-maker (DM) must choose one alternative *x* from a set *X*.
  - Chooses to maximize a utility function u.
  - u specifies how much utility DM gets from each alternative: $u : X \to \mathbb{R}$
- Example: DM chooses whether to eat an apple or a banana.

$$X = \{apple, banana\}$$

- Utility function might say *u* (*apple*) = 7, *u* (*banana*) = 12.
- Observe that we already started to use mathematics – but only language.

What do Utility Levels Mean? Hedonic Interpretation

- Utility is an objective measure of individual’s well-being.

*Nature has placed mankind under the governance of two sovereign masters, pain and pleasure. It is for them alone to point out what we ought to do. . . By the principle of utility is meant that principle which approves or disapproves of every action whatsoever according to the tendency it appears to have to augment or diminish the happiness of the party whose interest is in question: or, what is the same thing in other words to promote or to oppose that happiness. I say of every action whatsoever, and therefore not only of every action of a private individual, but of every measure of government. (*Jeremy Bentham*)*

•“ u (*apple*) = 7, u (*banana*) = 12” = apple gives 7 units of pleasure, banana gives 12 units of pleasure.

•This is not the standard way economists think about utility.

What do Utility Levels Mean? Revealed-Preference Interpretation

- Utility represents an individual's choices.
  - Individual choices are primitive data that economists can observe.
  - Choices are taken to reveal individual's preferences.
  - Utility is a convenient mathematical construction for modeling choices and preferences.

"u (*apple*) = 7, u (*banana*) = 12" = individual prefers bananas to apples.

"u (*apple*) = 2, u (*banana*) = 15" = individual prefers bananas to apples.

- How does this work?

Choice

- How can an individual's choices reveal her preferences?

- A choice structure (or choice dataset) $(\mathcal{B}, C\ )$ consists of:

1. A set $\mathcal{B}$ of choice sets $B \subseteq X$.

2. A choice rule C that maps each $B \in \mathcal{B}$ to a non-empty set of chosen alternatives $C(B) \subseteq B$.

    - C is a correspondence.

    - Interpretation: $C\ (B)$ is the set of alternatives the DM might choose from B.

Preference

- Goal: relate observable choice data to preferences over X.

- A preference relation $\succcurlyeq$ is a binary relation on X.
- "$x \succcurlyeq y$" means 'x is weakly preferred to y.'
- Given preference relation $\succcurlyeq$, define:
- Strict preference ($\succ$): $x \succ y \Leftrightarrow x \succcurlyeq y$ but not $y \succcurlyeq x$.
- Indifference ($\sim$): $x \sim y \Leftrightarrow x \succcurlyeq y$ and $y \succcurlyeq x$.
- Think a little bit about logic and set theory here.

Rational Preferences

- To make any progress, need to impose some restrictions on preferences.
- Most important: rationality

•Definition (MWG Definition 1B1)

- A preference relation $\succcurlyeq$ is rational if it satisfies:

1. Completeness: for all x, y, $x \succcurlyeq y$ or $y \succcurlyeq x$.
2. Transitivity: for all x, y, z, if $x \succcurlyeq y$ and $y \succcurlyeq z$, then $x \succcurlyeq z$.

- If $\succcurlyeq$ is rational, then $\succ$ and $\succcurlyeq$ are also transitive. (Prove this!)

- Hard to say much about behavior of irrational DM.

Maximizing a Preference Relation

- Optimal choices according to $\succcurlyeq$:
  - $C^*(B,t) = \{x \in B : x \succcurlyeq y \, for\, all\, y \in B\}$
  - $\succcurlyeq$ rationalizes choice data $(\mathcal{B}, C)$ if $C(B) = C^*(B,t)$ for all $B \in \mathcal{B}$

Fundamental Question of Revealed Preference Theory

- **When does choice data reveal that individual is choosing according to rational preferences?**

- **Definition**

- Given choice data $(\mathcal{B}, C)$, the revealed preference relation $\succcurlyeq^*$ is defined by $x \succcurlyeq^* y \Leftrightarrow$ there is some $B \in \mathcal{B}$ with $x, y \in B$ and $x \in C(B)$

- x is **weakly revealed** preferred to y if x is ever chosen when y is available. Notice that this allows for $y \in C(B)$ as one may have $x \sim y$.

- x is **strictly revealed preferred** to y if there is some $B \in \mathcal{B}$ with $x, y \in B$, $x \in C(B)$, and $y \notin C(B)$.

WARP

- Key condition on choice data for $\succcurlyeq^*$ to be rational and generate observed data: **weak axiom of revealed preference (WARP)**.

  - **Definition**

  - Choice data $(\mathcal{B}, C)$ satisfies WARP if whenever there exists $B \in \mathcal{B}$ with $x, y \in B$ and $x \in C(B)$, then for all $B' \in \mathcal{B}$ with $x, y \in B'$, it is not the case that both $y \in C(B')$ and $x \notin C(B')$.

  - "If *x* is weakly revealed preferred to *y*, then *y* cannot be strictly revealed preferred to *x*."

WARP: Example

- $X = \{x, y, z\}, \mathcal{B} = \{\{x, y\}, \{x, y, z\}\}$
- Choice rule C_1: $C_1(\{x, y\}) = \{x\}, C_1(\{x, y, z\}) = \{x\}$.
- Satisfies WARP: *x* is weakly revealed preferred to *y* and *z*, nothing is strictly revealed preferred to *x*.
- Choice rule C_2: $C_2(\{x, y\}) = \{x\}, C_2(\{x, y, z\}) = \{x, y\}$.
- Violates WARP: *y* is weakly revealed preferred to *x*, *x* is strictly revealed preferred to *y*.
- This is Exercise 1.C.1 (MWG).

A Fundamental Theorem of Revealed Preference

- **Theorem**

- *If choice data* $(\mathcal{B}, C\ )$ *satisfies WARP and includes all subsets of* X *of up to 3 elements, then* $\succcurlyeq^*$ *is rational and rationalizes the data: that is,* $C^*(B, \succcurlyeq^*) = C\ (B)$*. Furthermore, this is the only preference relation that rationalizes the data.* (MWG Proposition 1.D.2 – check book for proof)

- *Conversely, if the choice data violates WARP, then it cannot be rationalized by any rational preference relation.* (MWG Proposition 1.D.1) **PROOF HERE.**

A Fundamental Theorem of Revealed Preference

- *For the first part:* Remember weird B is a set of sets: this condition states that it must include all sets of up to three elements.

- Check MWG example 1D1 to see that we cannot drop this assumption.

- **Theorem tells us how individual's choices reveal her preferences: as long as choices satisfy WARP, can interpret choices as resulting from maximizing a rational preference relation.**

- We may conclude that if weird B includes all subsets of X, then choice and preferences work together just fine.

- But this is too restrictive: think of budget sets. We'll come back to this.

Preference and Utility

• Now that know how to infer preferences from choice, next step is representing preferences with a utility function.

•**Definition**

•A utility function $u : X \to \boldsymbol{R}$ represents preference relation $\succcurlyeq$ if, for all x, y, $x \succcurlyeq y \Leftrightarrow u(x) \geq u(y)$

•*banana* $\succcurlyeq$ *apple* is represented by both $u(apple) = 7, u(banana) = 12$ and $u(apple) = 2, u(banana) = 15$.

•If *u* represents $\succcurlyeq$, so does any strictly increasing transformation of u.

•Representing a given preference relation is an ordinal property.

•The numerical values of utility are cardinal properties.

What Preferences have a Utility Representation?

- **Theorem**
- *Only rational preferences relations can be represented by a utility function.* (MWG Proposition 1B2) **PROOF HERE – page 1.**
- *Conversely, if X is finite, any rational preference relation can be represented by a utility function.* (MWG exercise 1B5 - 'X finite' is only one possibility)

What Goes Wrong with Infinitely Many Alternatives?

- Lexicographic preferences: dictionary system – e.g., ‘I’m not going by plane”.

$$X = [0,1] \times [0,1]$$

- $(x_1, x_2) \succcurlyeq (y_1, y_2)$ if either
  - $x_1 > y_1$ or
  - $x_1 = y_1$ and $x_2 \geq y_2$

- Maximize first component. In case of tie, maximize second component.

Theorem

•*Lexicographic preferences cannot be represented by a utility function.* This is on MWG page 46. **PROOF HERE – page 3.**

Continuous Preferences

- What if rule out discontinuous preferences?

Definition

- For $X \subseteq \mathbf{R}^n$, preference relation $\succcurlyeq$ is continuous if whenever $x^m \to x$, $y^m \to y$, and $x^m \succcurlyeq y^m$ for all m, we have $x \succcurlyeq y$.

- Lexicographic preferences are not continuous: see example 3C1 cont.

PROOF HERE – page 4.

Continuous Preferences

Theorem

- *For $X \subseteq \mathbf{R}^n$, any continuous, rational preference relation can be represented by a* (continuous) *utility function.* This is MWG Proposition 3.C.1 – a bit advanced, but it's a good exercise if you like theory. **PROOF HERE – page 5.**

- Notice that this is very general. Assumptions are not very restrictive.

Review of Revealed Preference Theory

- If choice data satisfies WARP, can interpret as resulting from maximizing a rational preference relation.
- If set of alternatives is finite or preferences are continuous, can represent these preferences with a utility function.
- Utility function is just a convenient mathematical representation of individual’s ordinal preferences.
- Utility may or may not be correlated with pleasure/avoidance of pain.

Properties of Preferences and Utility Functions

- Doing useful analysis entails making assumptions.

- Try to do this carefully: make clearest, simplest, least restrictive assumptions.

- Understand what assumptions about utility correspond to in terms of preferences, since utility is just a way of representing preferences.

- We now cover some of the most important assumptions on preferences. (And, implicitly, on choices.)

Setting/Notation

- For rest of lecture, assume $X \subseteq \boldsymbol{R}^n$.
- Example: Consumer Problem: given fixed budget, choose how much of n goods to consume
  - Notation: for vectors $x = (x_1, \dots, x_n)$ and $y = (y_1, \dots, y_n)$,
  - $x \geq y$ means $x_k \geq y_k$ for all $k = 1, \dots, n$
  - $x > y$ means $x_k \geq y_k$ for all k and $x_k > y_k$ for some k
  - $x \gg y$ means $x_k > y_k$ for all k
- For example, (2,3,4) > (3,3,4) and (4,4,5)>>(3,3,4)

Setting/Notation

- For $\alpha \in [0, 1]$, $\alpha x + (1 - \alpha)\, y = (\alpha x_1 + (1 - \alpha)\, y_1, \ldots, \alpha x_n + (1 - \alpha)\, y_n)$

- This is a convex combination for each coordinate.

Monotonicity: Preferences

•"All goods are desirable"

Definition (MWG 3B2)

$\succcurlyeq$ is monotone if $x \geq y$ implies $x \succcurlyeq y$.

$\succcurlyeq$ is strictly monotone if $x > y$ implies $x \succ y$.

For example, strict monotonicity implies $(2,3,4) \succ (1,3,4)$.

Monotonicity: Utility

- If preferences are monotone, what does that mean for the utility function?

Theorem

Suppose utility function u represents preferences $\succcurlyeq$. Then:

u is non-decreasing $\Leftrightarrow$ $\succcurlyeq$ is monotone

u is strictly increasing $\Leftrightarrow$ $\succcurlyeq$ is strictly *monotone*

This is MWG Exercise 3B1. Let's show this. PROOF – page 7.

Local-Nonsatiation

"No bliss points." (Not even local ones.)

Let $B_\varepsilon(x) = \{y : \|x - y\| < \varepsilon\}$.

Definition

$\succcurlyeq$ is locally non-satiated if for any x and $\varepsilon > 0$, there exists $y \in B_\varepsilon(x)$ with $y \succ x$.

If u represents $\succcurlyeq$, then $\succcurlyeq$ is locally non-satiated iff u has no local maximum.

- This is MWG Figure 3B1, Definition 3B3.

Convexity

"Diversity is good."

Definition

$\succcurlyeq$ is convex if $x \succcurlyeq y$ and $x' \succcurlyeq y$ imply

$\alpha x + (1 - \alpha)\, x' \succcurlyeq y$ for all $\alpha \in (0, 1)$

$\succcurlyeq$ is strictly convex if $x \succcurlyeq y$, $x' \succcurlyeq y$, and $x \neq x'$ imply

- $\alpha x + (1 - \alpha)\, x' \succcurlyeq$ for all $\alpha \in (0, 1)$
- Does it make sense? $\frac{1}{2} beer + \frac{1}{2} wine$
- Turn now to properties of convex preferences

Contour Sets

- For $x \in X$, the upper contour set of x is

$$U(x) = \{y \in X : y \succcurlyeq x\}$$

(This is not the utility function!)

Theorem

$\succcurlyeq$ is convex iff $U(x)$ is a convex set for every $x \in X$. **PROOF HERE – PAGE 8**.

That's why convex preferences are called convex: for every x, the set of all alternatives preferred to x is convex.

Set of Maximizers

Theorem

- *If* t *is convex, then for any convex choice set B, the set* $C^*(B, t)$ *is convex.*
- *If* t *is strictly convex, then for any convex choice set B, the set*
- $C^*(B, \mathrm{t})$ *is single-valued (or empty).* Hence it's a function – **PAGE 9**.

Convexity: Utility Functions

- The characteristic of utility functions that represent convex preferences is quasi-concavity.

- **Definition**

- A function $u : X \to \mathbb{R}$ is **quasi-concave** if, for every x, y with $u(x) \geq u(y)$ and every $\alpha \in (0, 1)$,

$$u(\alpha x + (1 - \alpha) y) \geq u(y).$$

- A function $u : X \to \mathbb{R}$ is **strictly quasi-concave** if, for every x, y with $u(x) \geq u(y)$ and $x \neq y$ and every $\alpha \in (0, 1)$,

$$u(\alpha x + (1 - \alpha) y) > u(y).$$

- *Exercise:* show that u is quasi-concave iff, for every $r \in \mathbf{R}$, the upper contour set $\{x \in X : u(x) \geq r\}$ is convex.

Convexity: Utility Functions

- **Theorem**
- *Suppose utility function u represents preferences* t*. Then:*

u is quasi-concave $\Leftrightarrow \succcurlyeq$ *is convex*

u is strictly quasi-concave $\Leftrightarrow \succcurlyeq$ *is* strictly *convex*

- Warning: *convex* preferences are represented by quasi-*concave* utility functions.
- Convex preferences get that name because they make upper contour sets convex.
- Quasi-concave utility functions get that name because quasi-concavity is a weaker property than concavity.

Separability

- Often very useful to restrict ways in which a consumer’s preferences over one kind of good can depend on consumption of other goods.

- If allowed arbitrary interdependencies, would need to observe consumer’s entire consumption bundle to infer anything.

- Properties of preferences that separation among different kinds of goods are called separability properties.

Weak Separability: Preferences

"Preferences over one kind of goods don't depend on what other goods are consumed."

- Let $J_1, \ldots J_m$ be a list of m mutually exclusive subsets of the set of goods.
- Let J_k^C be the complement of J_k.
- Given a vector x, let x_{Jk} be the vector of those goods in J_k.

Definition

- $\succcurlyeq$ is weakly separable in $J_1, \ldots, J_m$ if, for every $k \in 1, \ldots, m$, every $x_{Jk}, x'_{Jk} \in \mathbf{R}|^{Jk}|$, and every $x_JkC, x'_JkC \in$

$\mathbf{R}|^{JkC}|$,

$$\left(x_{J_k}, x_{J_k^C}\right) \succsim \left(x'_{J_k}, x_{J_k^C}\right) \Leftrightarrow \left(x_{J_k}, x'_{J_k^C}\right) \succsim \left(x'_{J_k}, x'_{J_k^C}\right)$$

Weak Separability: Preferences

- Ex. $X = \{food, clothing, housing\}$, $m = 1$, $J_1 = \{food\}$ $\Longrightarrow$ preferences separable in food, not separable in clothing or housing.

Weak Separability: Utility

Theorem

Suppose utility function u represents preferences t*. Then* t *is weakly separable in* $J_1, \ldots, J_m$ *iff has utility representation of form:*

$$u(x) = v\left(u_1(x_{J_1}), \ldots, u_m(x_{J_m}), x_{(J_1 \cup \ldots \cup J_m)^C}\right).$$

"Food utility function" u_1, total utility is function of (food utility, clothing, housing).

Other Kinds of Separability

- “Strongly separable” preferences imply existence of additively separable utility:
  - m
  - $u\ (x) = \sum u_k\,(x_{Jk}\,)\ .$
  - $k\ {=}1$
- “No wealth effects in good 1” imply existence of quasi-linear utility:
  - $u\ (x) = x_1 + v\ (x_2, \ldots, x_n\,)\ .$
- Good 1 is called a numeraire (or “money”).