

# Externalities and Public Goods

## Simple Bilateral Externality

**Definition:** An externality is present whenever the well-being of a consumer or the production possibilities of a firm are **directly** affected by the actions of another agent in the economy.

Externalities may be positive or negative.

"Directly" exclude any effects that are mediated by prices.

That is, an externality is present if, say, a fishery's productivity is affected by the emissions from a nearby oil refinery, but not simply because the fishery's profitability is affected by the price of oil (which, in turn, is to some degree affected by the oil refinery's output of oil).

The latter type of effect [pecuniary externality] is present in any competitive market but creates no inefficiency.

Indeed, with price-taking behavior, the market is precisely the mechanism that guarantees a Pareto optimal outcome.

This suggests that the presence of an externality is not merely a technological phenomenon **but also a function of the set of markets in existence.**

Consider **partial** equilibrium model: **no income effect.**

Two consumers,  $i = 1, 2$ , who constitute a **small** part of the overall economy.

The actions of these consumers **do not affect the prices**  $p \in \mathbb{R}^L$  of the  $L$  traded goods in the economy.

At these prices, consumer  $i$ 's wealth is  $w_i$ .

Each consumer has preferences not only over her consumption of the  $L$  traded goods  $(x_{1i}, \dots, x_{Li})$  **but also over some action  $h \in \mathbb{R}_+$  taken by consumer 1.**

Consumer  $i$ 's (differentiable) utility function takes the form  $u_i(x_{1i}, \dots, x_{Li}, h)$

Assume  $\partial u_2(x_{12}, \dots, x_{L2}, h) / \partial h \neq 0$ .

Because consumer 1's choice of  $h$  affects consumer 2's well-being, it generates an externality.

For example, the two consumers may live next door to each other, and  $h$  may be a measure of how loudly consumer 1 plays music.

Or the consumers may live on a river, with consumer 1 further upstream. In this case,  $h$  could represent the amount of pollution put into the river by consumer 1; more pollution lowers consumer 2's enjoyment of the river.

Define for each consumer  $i$  a derived utility function over the level of  $h$ , assuming optimal commodity purchases by consumer  $i$  at prices  $p \in \mathbb{R}^L$  and wealth  $w_i$  :

$$v_i(p, w_i, h) = \text{Max}_{x_i \geq 0} \quad u_i(x_i, h) \\ \text{s.t. } p \cdot x_i \leq w_i$$

Assume that the consumers' utility functions take a **quasilinear** form with respect to a numeraire commodity.

Derived utility function  $v_i(\cdot)$  as  $v_i(p, w_i, h) = \phi_i(p, h) + w_i$

Since prices of the  $L$  traded goods are assumed to be unaffected by any of the changes we are considering, we shall suppress the price vector  $p$  and simply write  $\phi_i(h)$ .

We assume that  $\phi_i(\cdot)$  is twice differentiable with  $\phi_i''(\cdot) < 0$ .

Everything we do here applies if agents are firms (or one firm and one consumer).

## Nonoptimality of the Competitive Outcome

Consider a competitive equilibrium in which commodity prices are  $p$ .

That is, in equilibrium, each of the two consumers maximizes her utility limited only by her wealth and the prices  $p$  of the traded goods.

Consumer 1 chooses her level of  $h \geq 0$  to maximize  $\phi_1(h)$ .

Assume throughout interior solutions: now,  $h^* > 0$ .

The **equilibrium level** of  $h, h^*$ , satisfies the necessary and sufficient first-order condition

$$\phi_1'(h^*) = 0$$

Pareto optimal allocation: maximize the joint surplus of the two consumers:

$$\text{Max}_{h \geq 0} \phi_1(h) + \phi_2(h)$$

Necessary and sufficient first-order condition for  $h^\circ$  (assume strictly positive):

$$\phi_1'(h^\circ) = -\phi_2'(h^\circ)$$

When external effects are present, so that  $\phi_2'(h) \neq 0$  at all  $h$ , the equilibrium level of  $h$  is not optimal (unless  $h^\circ = h^* = 0$ ).

If  $\phi_2'(\cdot) < 0$  (negative externality), then  $\phi_1'(h^\circ) = -\phi_2'(h^\circ) > 0 = \phi_1'(h^*)$

That is,  $\phi_1'(h^\circ) > \phi_1'(h^*)$

$\phi_1'$  decreasing implies  $h^* > h^\circ$ .

Analogously,  $\phi_2'(\cdot) > 0$  (positive externality) implies  $h^* < h^\circ$ .

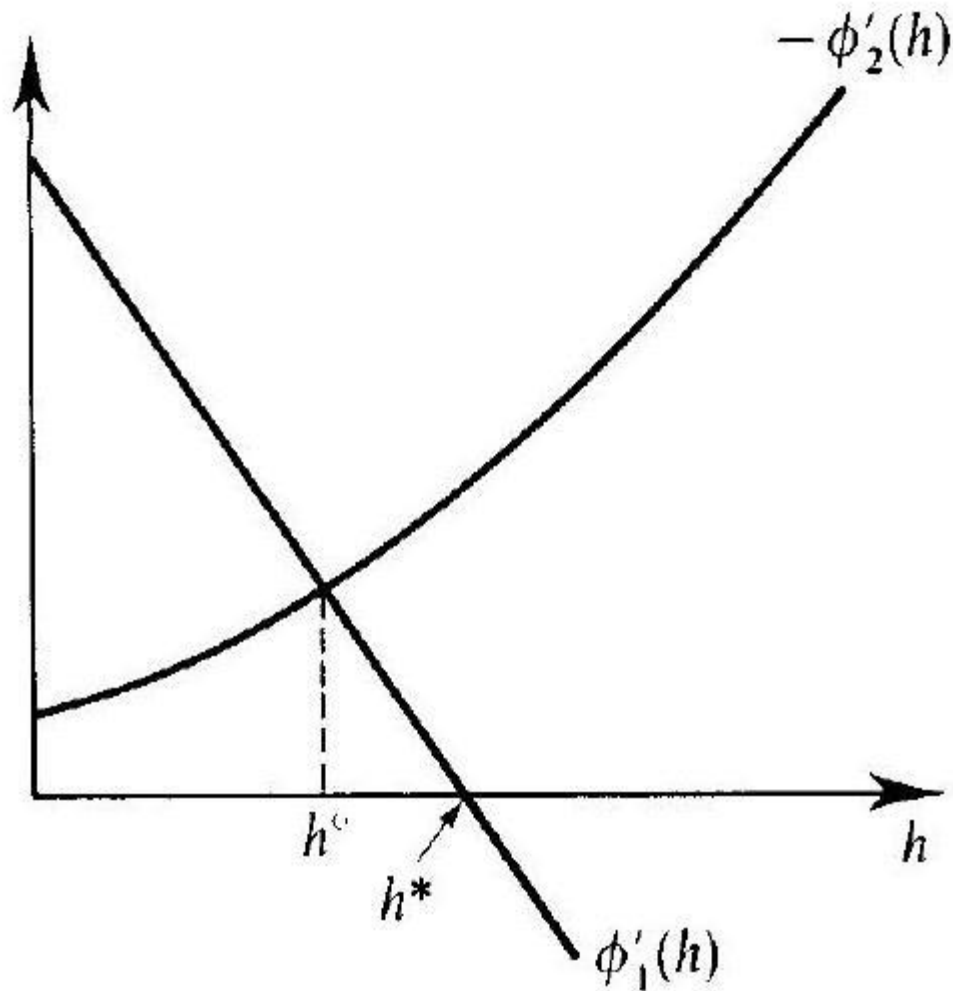


Figure 11.B.1: The equilibrium ( $h^*$ ) and optimal ( $h^\circ$ ) levels of a negative externality.

Figure 11.B.1 depicts the solution for a case in which  $h$  constitutes a negative external effect, so that  $\phi'_2(h) < 0$  at all  $h$ .

In the figure, we graph  $\phi'_1(\cdot)$  and  $-\phi'_2(\cdot)$ .

The competitive equilibrium level of the externality  $h^*$  occurs at the point where the graph of  $\phi'_1(\cdot)$  crosses the horizontal axis.

In contrast, the optimal externality level  $h^\circ$  corresponds to the point of intersection between the graphs of the two functions.

**Optimality does not usually entail the complete elimination of a negative externality.**

Externality's level is adjusted to the point where the marginal benefit to consumer 1 of an additional unit of the externality-generating activity,  $\phi'_1(h^\circ)$ , equals its marginal cost to consumer 2,  $-\phi'_2(h^\circ)$ .

In the current example, quasilinear utilities lead the optimal level of the externality to be **independent of the consumers' wealth levels**.

In the absence of quasi-linearity, wealth effects for the consumption of the externality make its optimal level depend on the consumers' wealth levels.

When the agents under consideration are firms, wealth effects are always absent.

## Traditional Solutions to the Externality Problem

### Quotas and taxes

Suppose that  $h$  generates a negative external effect, so that  $h^\circ < h^*$ .

Government can **mandate** that  $h$  be no larger than  $h^\circ$ , its optimal level.

With this constraint, consumer 1 will indeed fix the level of the externality at  $h^\circ$ .

Second option: tax on the externality-generating activity.

Pigouvian taxation.

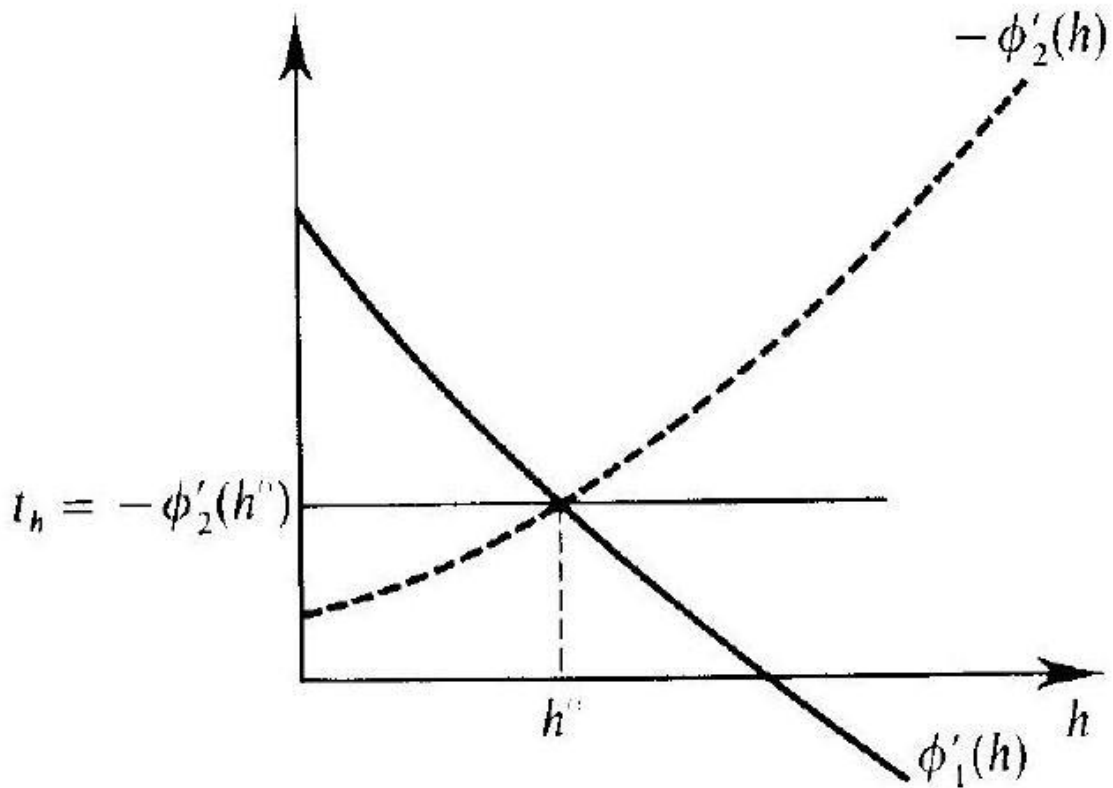


Figure 11.B.2: The optimality restoring Pigouvian tax.

Suppose that consumer 1 is made to pay a tax of  $t_h$  per unit of  $h$ .

Define  $t_h = -\phi'_2(h^*) > 0$

Consumer 1 will then choose the level of  $h$  that solves

$$\text{Max}_{h \geq 0} \phi_1(h) - t_h h$$

Necessary and sufficient first-order condition:

$$\phi'_1(h) = t_h$$

But  $t_h = -\phi'_2(h^*)$

Hence  $\phi'_1(h) = t_h = -\phi'_2(h)$

This is the condition for optimality.

Moreover, given  $\phi_1''(\cdot) < 0$ ,  $h^\circ$  must be the unique solution to problem.

Figure 11.B.2 illustrates this solution for a case in which  $h^\circ > 0$ .

**The optimality-restoring tax is exactly equal to the marginal externality at the optimal solution.**

That is, it is exactly equal to the amount that consumer 2 would be willing to pay to reduce  $h$  slightly from its optimal level  $h'$ .

When faced with this tax, consumer 1 is effectively led to carry out an individual cost benefit computation that internalizes the externality that she imposes on consumer 2.

Analogous for positive externality with  $t_h = -\phi_2'(h') < 0$

$t_h$  is a per-unit subsidy.

Comments about Pigouvian solution:

**First**, we can actually achieve optimality either by taxing the externality or by subsidizing its reduction.

Consider, for example, the case of a negative externality.

Suppose the government pays a subsidy of  $s_h = -\phi_2'(h^\circ) > 0$  for every unit that consumer 1's choice of  $h$  is below  $h^*$ , its level in the competitive equilibrium.

If so, then consumer 1 will maximize  $\phi_1(h) + s_h(h^* - h) = \phi_1(h) - t_h h + t_h h^*$ .

But this is equivalent to a tax of  $t_h$  per unit on  $h$  combined with a lump-sum payment of  $t_h h^*$ .

Hence, a subsidy for the reduction of the externality can replicate the outcome of the tax.

Distribution is different, but in principle may be combined with a lump-sum transfer

**Second**, in general, it is essential to tax the externality-producing activity directly.

Common example of this sort arises when a firm pollutes in the process of producing output.

A tax on its output leads the firm to reduce its output level but may not have any effect (or, more generally, may have too little effect) on its pollution emissions.

Taxing output achieves optimality only in the special case in which emissions bear a fixed monotonic relationship to the level of output.

In this special case, emissions can be measured by the level of output, and a tax on output is essentially equivalent to a tax on emissions.

**Third**, the tax/subsidy and the quota approaches are equally effective in achieving an optimal outcome.

However, the government must have a great deal of information about the benefits and costs of the externality to set the optimal levels of either the quota or the tax.

When the government does not possess this information the two approaches typically are not equivalent.

Fostering bargaining over externalities: enforceable property rights

Another approach to the externality problem aims at a less intrusive form of intervention:

Ensure that conditions are met **for the parties to reach an optimal agreement on the level of the externality**.

Suppose that we establish enforceable property rights with regard to the externality-generating activity.

For example, that we assign the right to an "externality-free" environment to consumer 2.

Then consumer 1 is unable to engage in the externality-producing activity without consumer 2's permission.



For simplicity, imagine that the bargaining between the parties takes a form in which consumer 2 makes consumer 1 a take-it-or-leave-it offer, demanding a payment of  $T$  in return for permission to generate externality level  $h$ .

Consumer 1 will agree to this demand if and only if she will be at least as well off as she would be by rejecting it.

That is, if and only if  $\phi_1(h) - T \geq \phi_1(0)$ .

Hence, consumer 2 will choose her offer  $(h, T)$  to solve

$$\begin{aligned} \text{Max}_{h \geq 0, T} \quad & \phi_2(h) + T \\ \text{s.t.} \quad & \phi_1(h) - T \geq \phi_1(0) \end{aligned}$$

The constraint is binding in any solution to this problem. In particular:

$$T = \phi_1(h) - \phi_1(0)$$

Therefore, consumer 2's optimal offer involves the level of  $h$  that solves

$$\text{Max}_{h \geq 0} \phi_2(h) + \phi_1(h) - \phi_1(0).$$

The solution is precisely  $h^\circ$ , the socially optimal level.

The precise allocation of these rights between the two consumers is inessential to the achievement of optimality.

Consumer 1 may have the right to generate as much of the externality as she wants.

In the absence of any agreement, consumer 1 will generate externality level  $h^*$ .

Now consumer 2 will need to offer a  $T < 0$  (i.e., to pay consumer 1) to have  $h < h^*$ .

In particular, consumer 1 will agree to externality level  $h$  if and only if:

$$\phi_1(h) - T \geq \phi_1(h^*).$$

As a consequence, consumer 2 will offer to set  $h$  at the level that solves:

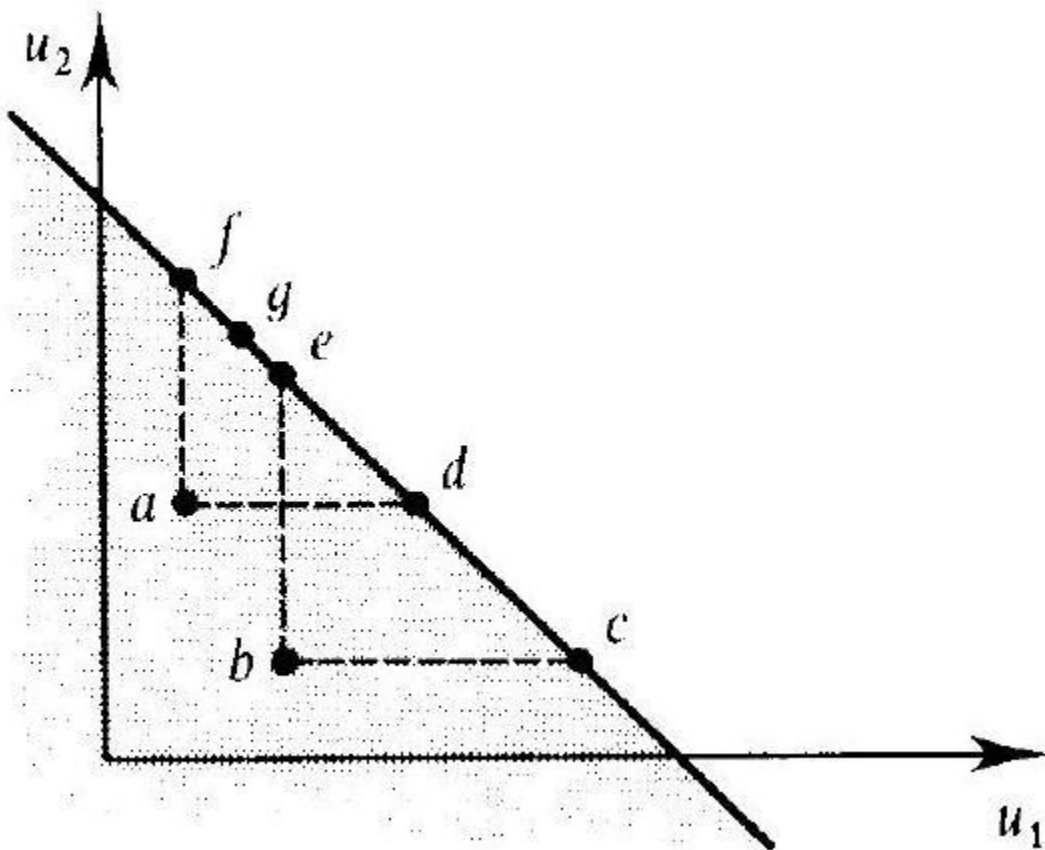
$$\text{Max}_h (\phi_2(h) + \phi_1(h) - \phi_1(h^*))$$

Once again, the optimal externality level  $h^\circ$  results.

**The allocation of rights affects only the final wealth of the two consumers by altering the payment made by consumer 1 to consumer 2.**

In the first case, consumer 1 pays  $\phi_1(h'') - \phi_1(0) > 0$  to be allowed to set  $h^\circ > 0$

In the second, she "pays"  $\phi_1(h'') - \phi_1(h^*) < 0$  in return for setting  $h^\circ < h^*$ .



*Figure 11.B.3: The final distribution of utilities under different property rights institutions and different bargaining procedures.*

This is an instance of what is known as the **Coase theorem** [for Coase (1960)]:

**If trade of the externality can occur, then bargaining will lead to an efficient outcome no matter how property rights are allocated.**

**Moreover, no income effect => efficiency and distribution problems can be separated: efficient level of externality does not depend on allocation of property rights.**

The existence of both well-defined and enforceable property rights is essential for this type of bargaining to occur.

If property rights are not well defined, it will be unclear whether consumer 1 must gain consumer 2's permission to generate the externality.

If property rights cannot be enforced (perhaps the level of  $h$  is not easily measured), then consumer 1 has no need to purchase the right to engage in the externality-generating activity from consumer 2.

For this reason, **proponents of this type of approach focus on the absence of these legal institutions as a central impediment to optimality.**

This solution to the externality problem has a significant advantage over the tax and quota schemes in terms of the level of knowledge required of the government.

The consumers must know each other's preferences, but the government need not.

For bargaining over the externality to lead to efficiency, it is important that the consumers know this information.

When the agents are to some extent ignorant of each others' preferences, bargaining need not lead to an efficient outcome.

Two further points:

**First**, in the case in which the two agents are firms, one form that an efficient bargain might take is the sale of one of the firms to the other.

The resulting merged firm would then fully internalize the externality in the process of maximizing its profits.

This conclusion presumes that the owner of a firm has full control over all its functions. In more complicated (but realistic) settings in which this is not true, say because owners must hire managers whose actions cannot be perfectly controlled, the results of a merger and of an agreement over the level of the externality need not be the same.

See Holmstrom and Tirole (1989) for a discussion of these issues in the theory of the firm.

**Second**, note that all three approaches require that the externality-generating activity be measurable.

This is not a trivial requirement; in many cases, such measurement may be either technologically infeasible or very costly (consider the cost of measuring air pollution or noise).

A proper computation of costs and benefits should take these costs into account. **If measurement is very costly, then it may be optimal to simply allow the externality to persist.**

## Externalities and Missing Markets

There is a **connection between externalities and missing markets**.

A market system can be viewed as a particular type of trading procedure (which is just a form of social interaction).

Suppose that property rights are well defined and enforceable and that a competitive market for the right to engage in the externality-generating activity exists.

For simplicity, assume that consumer 2 has the right to an externality-free environment.

Let  $p_h$  denote the price of the right to engage in one unit of the activity.

In choosing how many of these rights to purchase, say  $h_1$ , consumer 1 will solve

$$\text{Max}_{h_1 \geq 0} \phi_1(h_1) - p_h h_1$$

which has the first-order condition

$$\phi'_1(h_1) = p_h$$

In deciding how many rights to sell,  $h_2$ , consumer 2 will solve

$$\text{Max}_{h_2 \geq 0} \phi_2(h_2) + p_h h_2$$

which has the first-order condition

$$\phi'_2(h_2) = -p_h$$

In a competitive equilibrium, the market for these rights must clear:  $h_1 = h_2$ .

Hence, the level of rights traded in this competitive rights market satisfies

$$\phi'_1(h) = -\phi'_2(h)$$

This is the optimal level  $h = h^\circ$ .

The equilibrium price of the externality is  $p_h^* = \phi'_1(h^\circ) = -\phi'_2(h^\circ)$ .

Consumer 1 and 2's equilibrium utilities are then  $\phi_1(h^\circ) - p_h^* h^\circ$  and  $\phi_2(h^\circ) + p_h^* h^\circ$ , respectively.

**The market therefore works as a particular bargaining procedure for splitting the gains from trade.**

If a competitive market exists for the externality, then optimality results.

**Externalities can be seen as being inherently tied to the absence of certain competitive markets.**

Indeed, our definition of an externality explicitly required that an action chosen by one agent must directly affect the well-being or production capabilities of another.

Once a market exists for an externality, however, each consumer decides for herself how much of the externality to consume at the going prices.

The idea of a competitive market for the externality in the present example is rather unrealistic.

In a market with only one seller and one buyer, price taking would be unlikely.

However, most important externalities are produced and felt by many agents.

Thus, we might hope that in these multilateral settings, price taking would be a more reasonable assumption and, as a result, that a competitive market for the externality would lead to an efficient outcome.

## Public Goods

**Definition:** A public good is a commodity for which use of a unit of the good by one agent does not preclude its use by other agents.

Public goods are nondepletable:

Consumption by one individual does not affect the supply available for other individuals.

Knowledge provides a good illustration.

The use of a piece of knowledge for one purpose does not preclude its use for others.

Commodities studied up to this point have been assumed to be of a private, or depletable, nature;

A distinction can also be made according to whether exclusion of an individual from the benefits of a public good is possible.

**Every private good is automatically excludable, but public goods may or may not be.**

The patent system, for example, is a mechanism for excluding individuals (although imperfectly) from the use of knowledge developed by others.

On the other hand, it might be technologically impossible, or at the least very costly, to exclude some consumers from the benefits of national defense or of a project to improve air quality.

Focus here on the case in which exclusion is not possible.

A public "good" need not necessarily be desirable; that is, we may have public bads (e.g., foul air).

In this case, we should read the phrase "does not preclude" to mean "does not decrease."

## Conditions for Pareto Optimality

Consider a setting with  $I$  consumers and one public good, in addition to  $L$  traded goods of the usual, private, kind.

Partial equilibrium perspective: the quantity of the public good has no effect on the prices of the  $L$  traded goods and that each consumer's utility function is quasilinear with respect to the same numeraire, traded commodity.

We can therefore define, for each consumer  $i$ , a derived utility function over the level of the public good.

Let  $x$  denote the quantity of the public good.

Denote consumer  $i$ 's utility from the public good by  $\phi_i(x)$ .

Assume that this function is twice differentiable, with  $\phi_i''(x) < 0$  at all  $x \geq 0$ .

Precisely because we are dealing with a public good, the argument  $x$  does not have an  $i$  subscript.

The cost of supplying  $q$  units of the public good is  $c(q)$ .

Assume that  $c(\cdot)$  is twice differentiable, with  $c''(q) > 0$  at all  $q \geq 0$ .

Take  $\phi_i'(\cdot) > 0$  for all  $i$  and  $c'(\cdot) > 0$ : wlog, public good is desirable and costly.

In this quasilinear model, any Pareto optimal allocation must maximize aggregate surplus.

Therefore must involve a level of the public good that solves

$$\text{Max}_{q \geq 0} \sum_{i=1}^I \phi_i(q) - c(q)$$

The necessary and sufficient first-order condition for the optimal quantity  $q^\circ$  is then

$$\sum_{i=1}^I \phi_i'(q^\circ) = c'(q^\circ)$$

This condition is the classic optimality condition for a public good first derived by Samuelson (1954; 1955).

**At the optimal level of the public good the sum of consumers' marginal benefits from the public good is set equal to its marginal cost.**

For a private good, where each consumer's marginal benefit from the good is equated to its marginal cost.

## Inefficiency of Private Provision of Public Goods

Consider a public good provided by means of private purchases by consumers.

We imagine that a market exists for the public good and that each consumer  $i$  chooses how much of the public good to buy, denoted by  $x_i \geq 0$ , taking as given its market price  $p$ .

The total amount of the public good purchased by consumers is then  $x = \sum_i x_i$ .

Formally, we treat the supply side as consisting of a single profit-maximizing firm with cost function  $c(\cdot)$  that chooses its production level taking the market price as given.

(We can also think of the supply behavior of this firm as representing the industry supply of  $J$  price-taking firms whose aggregate cost function is  $c(\cdot)$ .)



At a competitive equilibrium involving price  $p^*$ , each consumer  $i$ 's purchase of the public good  $x_i^*$  must maximize her utility:

$$\text{Max}_{x_i \geq 0} \phi_i \left( x_i + \sum_{k \neq i} x_k^* \right) - p^* x_i$$

In determining her optimal purchases, consumer  $i$  takes as given the amount of the private good being purchased by each other consumer.

There is a bit of game theory here: this is how we find a Nash equilibrium.

Consumer  $i$ 's purchases  $x_i^*$  must therefore satisfy the necessary and sufficient first-order condition

$$\phi_i' \left( x_i^* + \sum_{k \neq i} x_k^* \right) = p^*$$

Letting  $x^* = \sum_i x_i^*$  denote the equilibrium level of the public good, for each consumer  $i$  we must therefore have (for  $x_i^* > 0$ )

$$\phi_i'(x^*) = p^*$$

The firm's supply  $q^*$ , on the other hand, must solve:

$$\text{Max}_{q \geq 0} (p^* q - c(q))$$

and therefore must satisfy the standard necessary and sufficient first-order condition

$$p^* = c'(q^*)$$

At a competitive equilibrium,  $q^* = x^*$

Hence:

$$\phi_i'(q^*) = p^* = c'(q^*)$$

Or simply:

$$\phi_i'(q^*) = c'(q^*)$$

Hence:

$$\sum_i [\phi'_i(q^*) - c'(q^*)] = 0$$

Recalling that  $\phi'_i(\cdot) > 0$  and  $c'(\cdot) > 0$ , this implies that whenever  $I > 1$  and  $q^* > 0$  we have

$$\sum_{i=1}^I \phi'_i(q^*) > c'(q^*)$$

Whenever  $q^0 > 0$  and  $I > 1$ , the level of the public good provided is too low; that is,  $q^* < q^0$ .

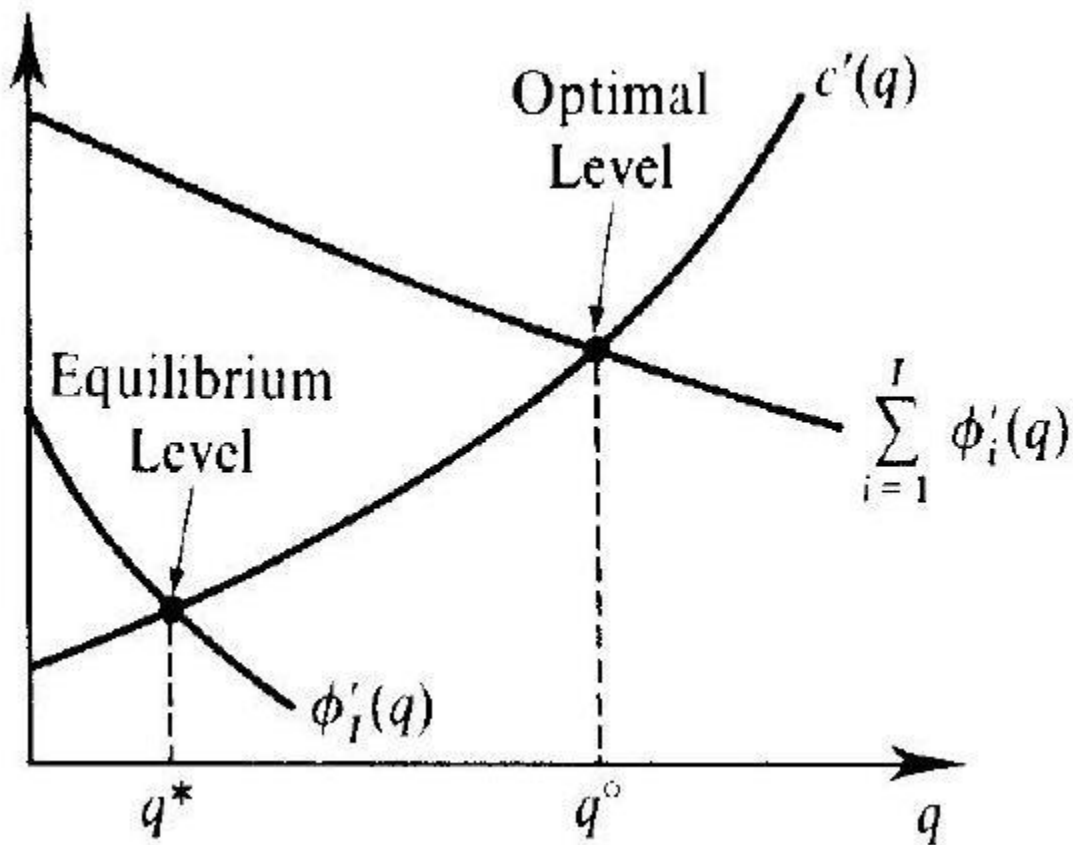


Figure 11.C.1: Private provision leads to an insufficient level of a desirable public good.

The cause of this inefficiency can be understood in terms of our discussion of externalities.

Each consumer's purchase of the public good provides a direct benefit not only to the consumer herself but also to every other consumer.

Hence, private provision creates a situation in which externalities are present.

The failure of each consumer to consider the benefits for others of her public good provision is often referred to as the free-rider problem:

**Each consumer has an incentive to enjoy the benefits of the public good provided by others while providing it insufficiently herself.**

In the present model, the free-rider problem takes a very stark form.

To see this most simply, suppose that we can order the consumers according to their marginal benefits, in the sense that  $\phi'_1(x) < \dots < \phi'_I(x)$  at all  $x \geq 0$ .

Then optimality can hold with equality only for a single consumer and, moreover, this must be the consumer labeled  $I$ .

**Therefore, only the consumer who derives the largest (marginal) benefit from the public good will provide it; all others will set their purchases equal to zero in the equilibrium.**

The equilibrium level of the public good is then the level  $q^*$  that satisfies  $\phi'_I(q^*) = c'(q^*)$ .

Figure 11.C.1 depicts both this equilibrium and the Pareto optimal level. Note that the curve representing  $\sum_i \phi'_i(q)$  geometrically corresponds to a vertical summation of the individual curves representing  $\phi_i(q)$  for  $i = 1, \dots, I$ .

(Whereas in the case of a private good, the market demand curve is identified by adding the individual demand curves horizontally).

The inefficiency of private provision is often remedied by governmental intervention in the provision of public goods.

Just as with externalities, this can happen not only through quantity-based intervention (such as direct governmental provision) but also through "price-based" intervention in the form of taxes or subsidies.

For example, suppose that there are two consumers with benefit functions  $\phi_1(x_1 + x_2)$  and  $\phi_2(x_1 + x_2)$ , where  $x_i$  is the amount of the public good purchased by consumer  $i$ , and that  $q^\circ > 0$ .

A **subsidy** to each consumer  $i$  per unit purchased of  $s_i = \phi'_{-i}(q^\circ)$  faces each consumer with the marginal external effect of her actions and so generates an optimal level of public good provision by consumer  $i$ .

Formally, if  $(\tilde{x}_1, \tilde{x}_2)$  are the competitive equilibrium levels of the public good purchased by the two consumers given these subsidies, and if  $\tilde{p}$  is the equilibrium price, then consumer  $i$ 's purchases of the public good,  $\tilde{x}_i$ , must solve:

$$\text{Max}_{x_i \geq 0} \phi_i(x_i + \tilde{x}_j) + s_i x_i - \tilde{p} x_i$$

and so  $\tilde{x}_i$  must satisfy the necessary and sufficient first-order condition

$$\phi'_i(\tilde{x}_1 + \tilde{x}_2) + s_i \leq \tilde{p}, \text{ with equality of } \tilde{x}_i > 0.$$

Substituting for  $s_i$ , and using the fact that price equals marginal cost and the market-clearing condition that  $\tilde{x}_1 + \tilde{x}_2 = \tilde{q}$ , we conclude that  $\tilde{q}$  is the total amount of the public good in the competitive equilibrium given these subsidies if and only if

$$\phi'_i(\tilde{q}) + \phi'_{-i}(q^\circ) \leq c'(\tilde{q})$$

with equality for some  $i$  if  $\tilde{q} > 0$ .

Use  $\sum_{i=1}^I \phi'_i(q^\circ) \leq c'(q^\circ)$  to see that  $\tilde{q} = q^\circ$ .

Note that both optimal direct public provision and this subsidy scheme require that the government know the benefits derived by consumers from the public good.

I.e., their willingness to pay in terms of private goods.

## Lindahl Equilibria

Although private provision of the sort studied above results in an inefficient level of the public good, there is in principle a market institution that can achieve optimality.

Suppose that, for each consumer  $i$ , we have a market for the public good "as experienced by consumer  $i$ ."

That is, we think of each consumer's consumption of the public good as a distinct commodity with its own market.

We denote the price of this personalized good by  $p_i$ .

Note that  $p_i$  may differ across consumers.

Suppose also that, given the equilibrium price  $p_i^{**}$ , each consumer  $i$  sees herself as deciding on the total amount of the public good she will consume,  $x_i$ , so as to solve

$$\text{Max}_{x_i > 0} \phi_i(x_i) - p_i^{**} x_i$$

Her equilibrium consumption level  $x_i^{**}$  must therefore satisfy the necessary and sufficient first-order condition

$$\phi_i'(x_i^{**}) \leq p_i^{**}, \text{ with equality if } x_i^{**} > 0$$

The firm is now viewed as producing a bundle of  $I$  goods with a fixed-proportions technology (i.e., the level of production of each personalized good is necessarily the same).

Thus, the firm solves

$$\text{Max}_{q \geq 0} \left( \sum_{i=1}^I p_i^{**} q \right) - c(q)$$

The firm's equilibrium level of output  $q^{**}$  therefore satisfies the necessary and sufficient first-order condition

$$\sum_{i=1}^I p_i^{**} \leq c'(q^{**}), \text{ with equality if } q^{**} > 0$$

We have then:

$$\sum_{i=1}^I \phi_i'(q^{**}) \leq c'(q^{**}), \text{ with equality if } q^{**} > 0$$

The equilibrium level of the public good consumed by each consumer is exactly the efficient level:  $q^{**} = q^{\circ}$ .

This type of equilibrium in personalized markets for the public good is known as a Lindahl equilibrium, after Lindahl (1919).

To understand why we obtain efficiency, note that once we have defined personalized markets for the public good, each consumer, taking the price in her personalized market as given, fully determines her own level of consumption of the public good.

**Externalities are eliminated.**

Yet, despite the attractive properties of Lindahl equilibria, their realism is questionable.

Note, first, that the ability to exclude a consumer from use of the public good is essential if this equilibrium concept is to make sense.

Otherwise a consumer would have no reason to believe that in the absence of making any purchases of the public good she would get to consume none of it.

Moreover, even if exclusion is possible, these are markets with only a single agent on the demand side.

As a result, price-taking behavior of the sort presumed is unlikely to occur.

The idea that inefficiencies can in principle be corrected by introducing the right kind of markets is a very general one.

In particular cases, however, this "solution" may or may not be a realistic possibility.