Market structure and knowledge acquisition by firms

September 23, 2019

PEDRO HEMSLEY (Corresponding Author)
Federal University of Rio de Janeiro
pedro.hemsley@ie.ufrj.br or pedrohemsley@gmail.com
Av. Pasteur, 250, s.119, Rio de Janeiro - RJ, Brazil, 22290-902

RAFAEL PINHO
State University of Rio de Janeiro

KARINNA RIBEIRO
State University of Rio de Janeiro

Abstract

This paper builds on the literature that opens the black-box of Firm Theory by assuming problems have to be solved for effective production to take place, and knowledge is an endogenous input for problem-solving. We characterize the relationship between the predictability of production processes and investment in knowledge within a simplified one-layer firm, across different market structures. We show that firms working closer to the production frontier (those with a larger efficient scale in perfect competition, facing a higher demand in monopoly or more competitive internationally in an open economy) react more in terms of investment in knowledge when problem predictability changes. Our results help understanding knowledge accumulation decisions by firms facing different competitive environments and how they are affected by the characteristics of the problems faced in production.

Keywords: knowledge accumulation; market structures; predictability of production processes.

JEL: D21, D41, D42, L11.

1 Introduction

The growing literature on knowledge-based hierarchies opens the black-box of Firm Theory. It assumes problems have to be solved for effective production to take place, and knowledge is the endogenous input for that. This short paper studies the relationship between the patterns of the (mass) distribution of those problems and investment in knowledge accumulation within a firm. We study the sensitivity of knowledge to changes in the predictability of the production process – a key parameter of the distribution of problems, taken as exogenous in the literature – and how such relationship is affected by the market structure (and by trade openness). Intuitively, a predictable production process is one for which unusual problems have low probability.[1]

Garicano (2000) has set the foundations of the literature by providing the basic model (of knowledge-communication-production) to study hierarchies in firms - that is, the optimal number of layers in its organizational structure. He uses implicitly a perfect-competition market structure and performs the same comparative statics we are interested in. However, the impact of predictability on knowledge (for each layer) is ambiguous in his setting. We depart from Garicano (2000) in two ways. First, we consider only one layer of workers so as to find the impact of predictability on total knowledge, and check how this impact depends on other parameters. Second, we assume a specific market structure at a time, which allows us to compare the impact of predictability on knowledge under different market structures.

We do that within a simplified version of the model in Caliendo and Rossi-Hansberg (2012), henceforth CR. CR embed the model developed by Garicano (2000) in what Antràs and Yeaple (2014) call "the workhorse monopolistic competition model with constant elasticity of substitution (CES) preferences" in order to study the relationship between trade and firm organization.[2] As our focus is not on trade (although we have a result on trade openness), but on knowledge-accumulation in different settings, we study the two extreme market structures: perfect competition and monopoly.

According to Garicano (2000): "Results on the relation between the predictability of the production process and aspects of organizational design were previously available only in Athey et al. (1994)". And Garicano (2000) is the only paper in this literature to explicitly care for the relationship we focus here, but finds an ambiguous result.[3]

Our main finding is that the intuitively expected monotonic relation holds: a complex production process (as opposed to a predictable one) leads to more investment in knowledge accumulation. However, the size of the impact of problem complexity on knowledge accumulation depends critically on the distance between the firm output and the production

[1] Predictable problems are also referred to as frequent or easy-to-solve, or simply easy problems in the literature. Unpredictable ones are called complex problems.

[2] On the trade side, see Feenstra (2015) for a presentation of recent models of international economics, including the role of firm organization. See also Gumpert (2018) for a recent analysis of knowledge as a costly input in multinational firms.

[3] In Athey et al. (1994), according to Garicano (2000): "the intrinsic value of a decision by different employees in different states of the world is different" and "more complexity implies under most circumstances more discretion for production workers and less discretion for supervisors (this is ambiguous in the analysis I present)". We — and the remainder of this knowledge-based hierarchies literature — follow Garicano (2000): "The analysis presented here does not impose a priori any difference in the intrinsic value of the performance of managers and workers in a given state. A more complex production process is simply one that puts more weight on unusual states." The remainder of the literature, however, takes that complexity as given. Garicano and Rossi-Hansberg (2015) present a thorough review of the literature, but do not mention changes in predictability. In Garicano and Rossi-Hansberg (2004) there is no knowledge acquisition (among ex-ante heterogeneous skilled agents), while in Garicano and Rossi-Hansberg (2006) knowledge is a function of (heterogeneous) workers' types decision on knowledge accumulation, but predictability is fixed. Garicano and Rossi-Hansberg (2012) builds a dynamic setting – focusing on how these hierarchies grow, but once again take predictability as given.

frontier. This holds for varied market structures and degrees of trade openness, but that size also changes accordingly.

We take an Industrial Organization perspective on the growing literature opening the black-box of Firm Theory. Moreover, we end up contributing to the vast literature on the microeconomic foundations of underdevelopment.[4,5] Section 2 presents our model and section 3 our results. Section 4 concludes this paper. The appendix collects detailed derivations of results in the main text.

2 Model

In CR, entrepreneurs choose the number of hierarchical/managerial layers and the knowledge and span of control of each manager (i.e. the number of employees in each layer), given the distribution of problems that may arise. A problem has to be solved for a production possibility to become output. More predictable problems are easier (and thus less costly) to solve, requiring less knowledge (and managerial layers) within a firm. While CR takes such distribution of problems as exogenous, we do the comparative statics of it on knowledge accumulation.

Our setting is CR's baseline model with only one layer (or $L = 0$ in their notation), i.e. each firm is just one individual entrepreneur or a collection of partners with identical knowledge or problem-solving abilities. Assuming only one layer means we do not deal with the internal structure of firms.[6]

We also depart from the monopolistic competition model with constant-elasticity-of-substitution (CES) preferences, common in trade models. We study the two extreme market structures: perfect competition and monopoly. Besides these two departures, everything else is identical to CR and the reader should refer to it for full details on the setup.

Problems arise at the production process according to a decreasing density distribution $F(z) = 1 - e^{-\lambda \cdot z}$, where λ is our parameter of interest: the larger it is, the more predictable the production process is. A realization z means that such production possibility will turn into A units of output if the knowledge previously acquired included z. Equivalently, we will say that if the knowledge acquired by the entrepreneur is z, the fraction of problems he or she solves is $F(z)$ and his/her expected output is $F(z)A$. But knowing how to solve

[4]For a thorough framing and early review of the topic, the reader should refer to Stiglitz (1989). Grossman and Helpman (2018) "examine several potential determinants of growth and inequality, such as the productivity of an economy's manufacturing operations, and its capacity for innovation, and its policies to promote R&D" We look at knowledge accumulation within firms - one step behind the item "productivity of an economy's manufacturing operations" - and it turns out to depend on firms' competitiveness, distance to the production frontier and firm size - important factors in underdevelopment theories.

[5]Besides the cruciality of the distance to the production frontier, another unexpected feature approaches us to this literature on underdevelopment: results depend on the size of the exported good price. We will turn to this later.

[6]Also, this means the number of workers per layer is not a choice variable here. As in CR, we assume one worker at the top layer, which is the only existing one in our setup.

problems in the interval $[0, z]$ costs $c \cdot z$.[7,8]

A firm produces:

$$q = A\left[1 - e^{-\lambda Z}\right] \tag{1}$$

where A is the maximum output it can possibly reach (i.e. the one it reaches if all problems are solved), or its technological frontier; and Z is the knowledge in the firm.

If either λ or Z is too low, production is tiny for any given A. In the limit of no knowledge ($Z = 0$) or very unusual (ie., difficult or unpredictable) problems affecting the firm ($\lambda = 0$), one has $q = 0$. Note that in the exponential formulation, λ and Z are indistinguishable, as what matters is the product λZ. However, λ is exogenous there and Z is chosen by the firm, and this is what motivated our study of the impact of changes in the frequency of problems on knowledge accumulated within a firm. We assume throughout the paper $\lambda > 0$ (otherwise there would be no production). We interpret a larger λ firm as one that faces easier problems and whose production is ceteris paribus closer to the frontier A. Accordingly, one may interpret a larger λ for a given firm as the result of some improvement in the production process that enhances problem-solving (for a given knowledge stock Z) and approaches it to the maximum possible output A without affecting its technological frontier.

By rearranging equation (1), we obtain the level of knowledge a firm will choose if it is to produce q units of expected output:

$$Z = \frac{1}{\lambda} ln\left(\frac{A}{A - q}\right) \tag{2}$$

Parameter λ has two effects on the level of knowledge the firm will choose. The first is direct and negative, and is immediate from equation (2): λ and Z are inversely proportional. A higher (exogenous) λ induces a smaller (chosen) Z, as the firm will acquire less costly knowledge when it faces predictable, easy-to-solve problems.

The indirect effect operates through the expected output q in (2) and is positive. A higher (exogenous) λ induces a higher (chosen) q, which is increasing in λ, generating in turn a higher Z. Through this indirect effect, thus, λ and Z are directly proportional. Predictable problems are cheaper to solve and thus generate a larger production, making output closer to the frontier A, where the marginal cost is higher for a given amount of knowledge. The firm then optimally chooses some more knowledge to achieve that larger production – or, in other words, the decrease in knowledge due to the direct effect discussed above is softened.

Given a fixed production cost $f > 0$ (as in CR), total cost becomes:

[7] Knowledge is not assumed cumulative in CR, meaning one could know z without learning the whole interval $[0, z]$. However, agents do so, and cumulativeness simplifies the exposition. According to CR (footnote 10): "The decreasing density of problems is just a normalization, since agents always choose to learn how to solve the most common problems first. The exponential specification is not essential but adds some tractability to the model." Note that knowledge Z here is the same as z_0^0 in CR, i.e the knowledge of the single worker in the only layer of the firm. This also equals the total knowledge of the firm (in the one-layer case), denoted in CR as Z_0^0 . The additional scripts are unnecessary in our setup.

[8] In CR, this cost is wcz, i.e. "learning one unit of knowledge requires c units of time of a teacher at wage w". In our simplified setup, taking $w = 1$ is without loss of generality.

$$TC(q) = cZ + f = c \underbrace{\left[\frac{1}{\lambda} ln\left(\frac{A}{A-q}\right)\right]}_{\text{Z}} + f$$

Marginal and average costs are given by the following expressions, respectively:[9]

$$MC(q) = \frac{c}{\lambda(A-q)} \tag{3}$$

$$AC(q) = \frac{1}{q}\frac{c}{\lambda} ln\left(\frac{A}{A-q}\right) + \frac{f}{q}$$

3 Results

We perform our analysis in the two extreme market structures -– perfect competition and monopoly -– and also extend the result to an open economy, taking the international price as given.

3.1 Perfect Competition

Under perfect competition (in autarky), one may impose a zero-profit condition, which boils down to $MC(q) = AC(q)$. This generates the following implicit solution for output q:[10]

$$q = (A-q)\left[ln\left(\frac{A}{A-q}\right) + \frac{f\lambda}{c}\right] \tag{4}$$

By using the implicit function theorem, the derivative of q with respect to λ may be computed as:

$$q_\lambda = \frac{(A-q)}{\frac{c}{f} ln\left(\frac{A}{A-q}\right) + f} \tag{5}$$

Since $q < A$, output q is increasing in λ: predictable problems (high λ) lead to a high output. The derivative of Z with respect to λ , computed from equation (2), equals:

$$Z_\lambda = \underbrace{-\frac{1}{\lambda^2} ln\left(\frac{A}{A-q}\right)}_{\text{I}} + \underbrace{\frac{1}{\lambda}\left[\frac{1}{\frac{c}{f}\cdot ln\left(\frac{A}{A-q}\right) + \lambda}\right]}_{\text{II}} \tag{6}$$

[9] We follow CR instead of Garicano (2000) in solving the firm's cost-minimization problem. However, in CR, firms choose knowledge among other things, as number of layers and workers per layer, and hence a given level of output may be generated with different combinations of inputs. With one layer, our equation (1), also present in CR, establishes a one-to-one relationship between firm knowledge Z and expected output q. Therefore, we can jump straight to using q as the choice variable of the firm in the remainder.

[10] A non-negative solution exists for all parameter values, and is strictly positive if $\frac{f\lambda}{c} > 0$. One may also check that the second-order condition is respected.

Informally, one may observe from (6) that if q is close to A, then Z_λ is large (in absolute value). Term I in equation (6) represents the direct negative impact of the predictability of problems (λ) on knowledge Z, while II is the indirect positive effect. If output q is near A, term II becomes small and term I goes to negative infinity: close to the production frontier, knowledge increases by a large amount if production problems become less predictable (the smaller λ is). However, when output is far from the frontier, both terms I and II disappear, and knowledge becomes nearly insensitive to the predictability of problems.

The discussion in the previous paragraph is based on an endogenous variable: the (expected) output q produced by the firm. We now state and formalize the behavior of the derivative Z_λ in terms of the exogenous parameter f, the firm's fixed cost.

Proposition 1. *For a large enough fixed cost of production, a one-layer firm in perfect competition will produce closer to its frontier, and the impact of production predictability on knowledge accumulation tends to minus infinity. For a low enough fixed cost, the individual firm's efficient scale in perfect competition becomes small at the same time as the impact of production predictability on knowledge vanishes.*

Proof. Proposition 1 means that as f increases, q is chosen close to A and the derivative of Z with respect to λ tends to minus infinity. As f approaches zero, both q and this derivative converge to zero. Notice initially that the derivative of q with respect to f is:

$$q_f = \frac{(A-q)^{\lambda}/_{c}}{ln\left(\frac{A}{A-q}\right) + {}^{f\lambda}/_{c}} \tag{7}$$

This is strictly positive since $A > q$. This is quite intuitive: the larger the fixed cost is, the higher the firm's efficient scale (where MC(q) equals AC(q)) will be .

Consider first what happens when $f \to 0$. Then (4) implies that $q \to 0$ and therefore $ln\left({}^{A}/_{(A-q)}\right) \to 0$. Then term I in (6) goes to zero. As for term II, compute initially:

$$\lim_{f\to 0} \frac{1}{f} ln\left(\frac{A}{A-q}\right) = lim_{f\to 0}\frac{q_f}{A-q}$$

The equality above is a direct application of l'Hôpital's rule. Plugging the expression for q_f and simplifying, this becomes:

$$\lim_{f\to 0} \frac{{}^{\lambda}/_{c}}{\ln\left(\frac{A}{A-q}\right) + {}^{f\lambda}/_{c}} = \infty$$

It follows that term II in (6) tends to zero. Since both terms I and II go to zero, the derivative of Z with respect to λ go to zero when $f \to 0$.

We turn now to the case $f \to \infty$. Then $q_f > 0$ implies $q \to A$: otherwise q would have an upper bound strictly below A and (4) would not hold for f large enough. Hence ${}^{A}/_{(A-q)} \to \infty$ and term I in (6) goes to $-\infty$. As for term II, a computation similar to the one above shows that:

$$\lim_{f\to\infty} \frac{1}{f} ln\left(\frac{A}{A-q}\right) = 0$$

Hence II goes to $1/\lambda^2$, which is finite. The derivative of Z with respect to λ then goes to minus infinity.

□

Proposition 1 characterizes the relationship between the predictability of problems and knowledge acquisition within one-layer firms and it turns out to be a monotonic and quite intuitive one. While there is a large negative impact when output is close to the production frontier (i.e., when the fixed cost is large), this effect vanishes when production is arbitrarily low (i.e., when the fixed cost is small).

A useful interpretation of this result relates to firm size. Consider two distinct economic sectors, both in perfect competition as described above. The technological frontier parameter A is the same in the two sectors, but the efficient scale output q is different between them due to a difference in some other parameter, such as f: firms are larger in one sector (with a higher efficient scale) and smaller in the other. Then the distance $A-q$ from equilibrium output to the frontier is higher in the sector with smaller firms. Our setting then implies that in smaller firms, the impact of the predictability of problems on knowledge acquisition is milder than in large firms.

3.2 Monopoly

We now consider that the same firm (holding the technology described above) is instead a monopolist. In this case, however, we have to impose more structure. Suppose such monopolist faces a linear demand function $P(q)=\alpha-\beta q$, for strictly positive α and β. The marginal revenue is thus given by $\alpha-2\beta q$. Equating it to the marginal cost, one obtains the following implicit expression for the profit-maximizing output:

$$2\beta q^2-(2\beta A+\alpha)q+\alpha A-\frac{c}{\lambda}=0 \tag{8}$$

We now compute the impact of the predictability of problems on knowledge as α changes: it is a parameter of demand that affects output and input choices, while a change in the fixed cost f does not affect the monopolist's decision as long as profit remains above zero. Notice that the discriminant of the quadratic equation (8) is strictly positive for all α, implying that a solution always exists. One may then compute the following derivatives:

$$q_\lambda=-\frac{\frac{c}{\lambda^2}}{4\beta q-(2\beta A+\alpha)} \tag{9}$$

$$q_\alpha=-\frac{A-q}{4\beta q-(2\beta A+\alpha)} \tag{10}$$

Then $q<A$ implies that these expressions are strictly positive if $\alpha>2A\beta$, an assumption that we maintain from now on: demand, measured by the parameter α, is large enough. One may check that this means we are working with the smaller root of (8), which implies $lim_{\alpha\to\infty}q=A$. That is, output approaches its upper bound as demand rises arbitrarily.[11,12]

[11]Hence the larger root of (8) does not respect the restriction $q\le A$ for α large enough and may be ignored.

[12]The smaller root is positive for $\alpha>c/A\lambda$, an assumption we make to ensure an interior solution. Since

We may now compute the derivative of Z with respect to λ:

$$Z_\lambda = \underbrace{-\frac{1}{\lambda^2} ln\left(\frac{A}{A-q}\right)}_{\text{I}} + \underbrace{\frac{1}{\lambda}\left(\frac{1}{A-q}\right)\left(\frac{-\frac{c}{\lambda^2}}{4\beta q-(2\beta A+\alpha)}\right)}_{\text{III}} \tag{11}$$

Consider initially the value of Z_λ when $\alpha = 2A\beta$:[13]

$$Z_\lambda = \frac{1}{\lambda^2}\left[\frac{1}{2} - ln\left(\frac{4A\sqrt{\beta}}{\sqrt{8\frac{c}{\lambda}}}\right)\right] \tag{12}$$

This is strictly negative if $c/\lambda < 2A^2\beta/e$, in which e is the base of the natural logarithm. Remember c/λ is an important ratio in the literature, defined by Garicano (2000) as the *net cost of acquiring knowledge*. If this net cost is low enough compared to A, then the term I in (11) does not approach zero: the impact of λ on Z is not so small (in absolute value) exactly because output is closer to the frontier A.

We study now what happens when α is large, so that $q \to A$. Compute:

$$\frac{d}{d\alpha}(Z_\lambda)\Bigg|_{\substack{\alpha\to\infty \\ (q\to A)}} = \left(\frac{1}{\lambda^2}\frac{1}{A-q}\frac{1}{(4\beta q-2\beta A-\alpha)^2}\left[4\beta q-2\beta A-\alpha+\frac{c}{\lambda}(1-2\beta A+\alpha)\right]\right)\Bigg|_{\substack{\alpha\to\infty \\ (q\to A)}}$$

$$= 2\beta A\left(1-\frac{c}{\lambda}\right) - \alpha\left(1-\frac{c}{\lambda}\right) + \frac{c}{\lambda} \tag{13}$$

This is negative and goes to minus infinity if $c/\lambda < 1$. Again, this is an upper bound on the net cost of acquiring knowledge. This condition requires thus the hazard rate λ to be larger than the learning cost c (per unit of knowledge Z). Hence when $\alpha \to \infty$ and thus $q \to A$, the impact of λ on Z becomes arbitrarily large (in absolute value).

We summarize this discussion in the following proposition.

Proposition 2. *Assume the net cost of acquiring knowledge is sufficiently small for a one-layer monopolist facing a linear demand. Then, the impact of production predictability on knowledge accumulation becomes larger (in absolute value) the larger the demand (shifter) is. Such impact reaches a strictly positive lower bound (again in absolute value) when demand is sufficiently small.*

Notice this Proposition 2 is equivalent to: assuming $c/\lambda < min\{1, 2A^2\beta/e\}$, the impact of λ on Z becomes arbitrarily large in absolute value when $\alpha \to \infty$, and achieves the strictly negative lower bound (12) when α is small.

---

we have also assumed $\alpha > 2A\beta$, one has: $\alpha > max\{2A\beta, c/A\lambda\}$. For simplicity, we take $2A\beta > c/A\lambda$, so that $2A\beta$ is the lower bound for α: the net cost of acquiring knowledge c/λ is not too large compared to A. We also assume profit is positive – that is, f is not too high.

[13] One may check that in this case output is $q = A - \frac{1}{4\sqrt{\beta}}\sqrt{8\frac{c}{\lambda}}$

3.3 Open Economy

To study an open economy in the simplest possible setting, we turn again to the perfect competition framework. The firm will equate its marginal cost (3) to the international exogenous price P (at least in the short-run) leading to the following expression for its output q:

$$q = A - \frac{c}{\lambda P} \tag{14}$$

Notice that when P increases, q approaches A. Following the same steps as in subsection 3.1, we compute the derivatives below:

$$q_\lambda = \frac{c}{\lambda^2 P}$$

$$q_P = \frac{c}{\lambda P^2}$$

$$Z_\lambda = \underbrace{-\frac{1}{\lambda^2} ln\left(\frac{A}{A-q}\right)}_{\text{I}} + \underbrace{\frac{1}{\lambda}\frac{1}{A-q}\left[\frac{c}{\lambda^2 P}\right]}_{\text{IV}} \tag{15}$$

Again, I represents de direct negative impact of predictability λ on knowledge Z. However, in an open economy, the positive impact - now denoted by IV - depends on the international price P, which is exogenous. Specifically, the higher P is, the lower the positive part becomes. Intuition is straightforward: a high international price increases output, moving the firm closer to the frontier A, as one can see from equation (14).

We may establish the following result for an open economy.

Proposition 3. *The higher the competitive advantage of national firms (that is, higher P), the larger will be the (negative) impact of predictability of production problems on knowledge accumulation.*

Proof. Proposition (3) means that for all λ, the derivative of Z with respect to λ diverges to minus infinity as P increases.

When $P \to \infty$, equation (14) implies that $q \to A$ and hence $ln\left(\frac{A}{A-q}\right) \to \infty$. Hence term I in (15) goes to negative infinity. As for term IV, one may compute:

$$lim_{P\to\infty}(A-q)P = \frac{c}{\lambda}$$

Hence term IV converges to $1/\lambda^2$, which is finite. Then the derivative of Z with respect to λ goes to minus infinity as P increases.

□

In words: the higher the competitive advantage of national firms, the higher will be the negative impact of production predictability on knowledge accumulation.

If P is below the autarky price, firms facing perfect competition in the domestic market go out of business as the new price is below the minimum of their average cost. The interesting case is when the international price is above the autarky price, not an uncommon situation: it describes exporting firms.[14] Then, output q increases when the firm chooses output to set its marginal cost equal to P. For a given production frontier A, the gap $A - q$ decreases, and this strengthens the impact of the predictability of problems on knowledge.

In other words, the higher the international price faced by national firms in a global market, the more sensitive knowledge accumulation will be to changes in the predictability of production problems. This implies that if production problems become harder (lower λ) -– that is, for exporting sectors facing more unpredictable problems -– and if the firm margin is high (i.e. large P), investment in knowledge will face a huge (positive) impact. If on the other hand the margin is small, investment in knowledge is less responsive, as the firm is far from the production frontier.

4 Final Remarks

This paper used a simplified version of the model in Caliendo and Rossi-Hansberg (2012) to study how a firm's investment in knowledge responds to changes in the predictability of the problems that affect its production process. We found that this impact is larger when output is close to the technological frontier, both under perfect competition and for a monopolist, and also in an open economy. This is a quite intuitive result: away from the frontier, costly knowledge is less attractive when production problems get harder to solve. When firms approach the exogenous frontier, a small decrease in the predictability of problems — so that they become harder to solve – leads the firm to increase by a large amount its acquisition of knowledge.

However, since output is endogenous in all settings, we set our results in terms of the relevant exogenous variable that affect output in each market structure: fixed cost of production (in perfect competition), demand shifter (in monopoly) and international price (in an open economy). The direct negative impact of the predictability of problems remains precisely the same no matter the market structure. The positive indirect impact (through the quantity produced) depends on the market structure assumed. However, it is always dominated by the direct impact, no matter the market structure and the exogenous variable analyzed. The higher the fixed cost (in perfect competition), the demand (in monopoly) or the international price (in an open economy), the closer a firm will be to the production frontier and therefore more sensitive will be its knowledge accumulation decision to changes in the predictability of problems.

Under perfect competition, whenever the fixed cost rises, so that equilibrium output increases, the firm approaches the technological frontier and there is a larger impact of the predictability of problems on knowledge acquisition. In fact, this holds for other factors that increase output, given the technological frontier. One has the following result for an open-economy firm under perfect competition: as the international price rises, output increases and the firm moves closer to the frontier, and therefore the impact of the predictability of

[14] Obviously, a national monopolist or domestic oligopolist facing an international price P smaller than its autarky monopoly or oligopoly price can (possibly) reduce its price to the international price P and still make positive economic profit. For this reason, we are focusing on firms facing perfect competition *ex ante*, as they are more constrained by international competition, since they made zero economic profit *ex ante*.

problems on knowledge acquisition increases. In short, opening the economy to trade tends to strengthen the intuitive result that more severe problems in production lead to more knowledge accumulation.

As concerns monopoly, we provide conditions under which its behavior is similar to that of firms under perfect competition. Importantly, however, we establish that the impact of the predictability of problems on knowledge does not necessarily approach zero even at the lowest possible output a single seller is willing to offer. The result in the other limit — when demand goes to infinity — is consistent with the results for perfect competition and an open economy.

In short, our results show that firms working closer to the production frontier (those with higher fixed cost in perfect competition, facing a higher demand in monopoly or more competitive internationally in an open economy) are those which react more in terms of investment in knowledge when production problems are softened or hardened. If underdeveloped countries are characterized by a prevalence of low-complexity production processes, an issue to be verified empirically, then our results imply that all else equal, firms close to the production frontier in these countries should increase investment in knowledge by a large amount when problems become more difficult. Future research will extend our analysis to a North-South general equilibrium context, contributing more explicitly to reach conclusions on the behaviour of knowledge accumulation in underdeveloped economies.

References

Pol Antràs and Stephen R Yeaple. Multinational firms and the structure of international trade. In *Handbook of International Economics*, volume 4, pages 55–130. Elsevier, 2014.

Susan Athey, Joshua S Gans, Scott Schaefer, and Scott Stern. *The Allocation of Decisions in Organisations*. University of New South Wales, School of Economics, 1994.

Lorenzo Caliendo and Esteban Rossi-Hansberg. The impact of trade on organization and productivity. *The Quarterly Journal of Economics*, 127(3):1393–1467, 2012.

Robert C Feenstra. *Advanced international trade: theory and evidence*. Princeton University Press, 2015.

Luis Garicano. Hierarchies and the organization of knowledge in production. *Journal of political economy*, 108(5):874–904, 2000.

Luis Garicano and Esteban Rossi-Hansberg. Inequality and the organization of knowledge. *American Economic Review*, 94(2):197–202, 2004.

Luis Garicano and Esteban Rossi-Hansberg. Organization and inequality in a knowledge economy. *The Quarterly Journal of Economics*, 121(4):1383–1435, 2006.

Luis Garicano and Esteban Rossi-Hansberg. Organizing growth. *Journal of Economic Theory*, 147(2):623–656, 2012.

Luis Garicano and Esteban Rossi-Hansberg. Knowledge-based hierarchies: Using organizations to understand the economy. *Annual Review of Economics*, 7(1):1–30, 2015.

Gene M Grossman and Elhanan Helpman. Growth, trade, and inequality. *Econometrica*, 86 (1):37–83, 2018.

Anna Gumpert. The organization of knowledge in multinational firms. *Journal of the European Economic Association*, 16(6):1929–1976, 2018.

Joseph E Stiglitz. Markets, market failures, and development. *The American Economic Review*, 79(2):197–203, 1989.

5 Appendix

This appendix presents detailed derivations of equations and results in the main text.

Derivation of Equation (4)

$$MC(q) = AC(q)$$

$$\frac{c}{\lambda(A-q)} = \frac{1}{q}\frac{c}{\lambda} ln\left(\frac{A}{A-q}\right) + \frac{f}{q}$$

Multiply both sides by ${}^{q\lambda}/_{c}$:

$$\frac{q}{(A-q)} = ln\left(\frac{A}{A-q}\right) + \frac{\lambda f}{c}$$

Multiply now by $A - q$ to get:

$$q = (A-q)\left[ln\left(\frac{A}{A-q}\right) + \frac{\lambda f}{c}\right]$$

This is equation (4).

Derivation of Equation (5)

Rearrange equation (4) to define the following function H:

$$H = q - (A-q)\left[ln\left(\frac{A}{A-q}\right) + \frac{\lambda f}{c}\right]$$

Compute:

$$\frac{\partial H}{\partial q} = 1 - \left[(-1)\cdot\left(ln\left(\frac{A}{A-q}\right) + \frac{f\lambda}{c}\right) + \underbrace{(A-q)\frac{A-q}{A}\frac{A}{(A-q)^2}}_{=1}\right]$$

$$= ln\left(\frac{A}{A-q}\right) + \frac{\lambda f}{c}$$

$$\frac{\partial H}{\partial \lambda} = -(A-q)\frac{f}{c}$$

The implicit function theorem now implies:

$$\frac{\partial q}{\partial \lambda} = -\frac{\partial H/\partial \lambda}{\partial H/\partial q} = \frac{(A-q)f/c}{ln\left(\frac{A}{A-q}\right) + \lambda f/c} = \frac{(A-q)}{(c/f)\cdot ln\left(\frac{A}{A-q}\right) + \lambda}$$

Derivation of Equation (6)

To simplify notation, denote the derivative of q with respect to λ by q_λ. Noticing that q depends on λ, equation (2) implies:

$$\frac{dZ}{d\lambda} = -\frac{1}{\lambda^2} ln\left(\frac{A}{A-q}\right) + \frac{1}{\lambda}\frac{\cancel{(A-q)}}{\cancel{A}}\frac{q_\lambda \cancel{A}}{(A-q)^{\cancel{2}}}$$

Plugging the expression for q_λ, one obtains:

$$\frac{dZ}{d\lambda} = -\frac{1}{\lambda^2} ln\left(\frac{A}{A-q}\right) + \frac{1}{\lambda}\frac{1}{\cancel{(A-q)}}\frac{\cancel{(A-q)}}{\left[(c/f)\cdot ln\left(\frac{A}{A-q}\right) + \lambda\right]}$$

This is equation (6).

Derivation of Equation (7)

Use again the function H defined above to compute:

$$\frac{\partial H}{\partial f} = -(A-q)\lambda/c$$

The implicit function theorem now implies:

$$q_f \stackrel{\text{def}}{=} \frac{\partial q}{\partial f} = -\frac{\partial H/\partial f}{\partial H/\partial q} = \frac{(A-q)\lambda/c}{ln\left(\frac{A}{A-q}\right) + \lambda f/c} > 0$$

Derivation of limits used in the proof of Proposition 1

We analyze limits of the expression:

$$\frac{ln\left(\frac{A}{A-q}\right)}{f}$$

This is indeterminate when f goes to either zero or infinity. To use L'Hôpital's rule, we differentiate both the numerator and the denominator with respect to f to get:

$$lim\frac{ln\left(\frac{A}{A-q}\right)}{f} = lim\frac{\cancel{(A-q)}}{\cancel{A}}\frac{q_f \cancel{A}}{(A-q)^{\cancel{2}}} = lim\frac{1}{(A-q)}q_f = lim\frac{1}{\cancel{(A-q)}}\left[\frac{\cancel{(A-q)}\lambda/c}{ln\left(\frac{A}{A-q}\right) + \lambda f/c}\right]$$

Derivation of Equation (8)

Marginal Revenue equals marginal cost:

$$\alpha - 2\beta q = \frac{c}{\lambda(A-q)}$$

Multiply both sides by $(A-q)$:

$$\alpha A - \alpha q - 2\beta A q + 2\beta q^2 = \frac{c}{\lambda}$$

Collect the terms in q:

$$2\beta q^2 - q(\alpha + 2\beta A) + \alpha A - \frac{c}{\lambda} = 0$$

Implications of $\alpha > 2A\beta$

First, this assumption implies that we are working with the smaller root of (8).

The derivatives in equations (9) and (10) are strictly positive if the common denominator is negative: $4\beta q - (2\beta A + \alpha) < 0$. Rearranging, one has:

$$q < \frac{A}{2} + \frac{\alpha}{4\beta} \tag{16}$$

Let Δ denote the discriminant of the quadratic equation (10). The roots are:

$$q = \frac{2\beta A + \alpha + \sqrt{\Delta}}{4\beta} = \frac{A}{2} + \frac{\alpha}{4\beta} + \frac{\sqrt{\Delta}}{4\beta}$$

$$q = \frac{2\beta A + \alpha - \sqrt{\Delta}}{4\beta} = \frac{A}{2} + \frac{\alpha}{4\beta} - \frac{\sqrt{\Delta}}{4\beta}$$

Hence only the smaller root respects (16).

Second, it implies that $lim_{\alpha\to\infty} q = A$.

We must compute the limit of the smaller root of equation (8):

$$\lim_{\alpha\to\infty} q(\alpha) = \lim_{\alpha\to\infty}\left[\frac{A}{2} + \frac{\alpha}{4\beta} - \frac{1}{4\beta}\sqrt{4A^2\beta^2 - 4A\beta\alpha + \alpha^2 + 8\beta\frac{c}{\lambda}}\right]$$

$$= \frac{A}{2} + \frac{1}{4\beta}\cdot\lim_{\alpha\to\infty}\left[\alpha - \sqrt{\alpha^2 - \underbrace{(4A\beta)}_{\text{m}}\alpha + \underbrace{(4A^2\beta^2 + 8\beta(c/\lambda))}_{\text{k}}}\right]$$

But for any k, m, one has:

$$\lim_{\alpha\to\infty}\left[\alpha - \sqrt{\alpha^2 - m\alpha + k}\right] = \frac{m}{2} = 2A\beta$$

Therefore:

$$\lim_{\alpha\to\infty} q(\alpha) = \frac{A}{2} + \frac{1}{4\beta}\cdot 2A\beta = A$$

Derivation of equations (9) and (10)

As before, let $H(q, \lambda, \alpha)$ denote the left-hand side of equation (8). Compute:

$$H_q = 4\beta q - (2\beta A + \alpha)$$
$$H_\alpha = A - q$$
$$H_\lambda = \frac{c}{\lambda^2}$$

The implicit function theorem implies:

$$q_\lambda = -\frac{c/\lambda^2}{4\beta q - (2\beta A + \alpha)}$$

$$q_\alpha = -\frac{A - q}{4\beta q - (2\beta A + \alpha)}$$

Derivation of equation (12)

We find initially the output q produced when $\alpha = 2A\beta$:

$$q = \left[\frac{A}{2} + \frac{\alpha}{4\beta} - \frac{1}{4\beta}\sqrt{4A^2\beta^2 - 4A\beta\alpha + \alpha^2 + 8\beta\frac{c}{\lambda}}\right]$$

$$= \frac{A}{2} + \frac{2a\beta}{4\beta} - \frac{1}{4\beta}\sqrt{4A^2\beta^2 - 8A^2\beta^2 + 4A^2\beta^2 + 8\beta c/\lambda}$$

$$= A - \frac{1}{4\beta}\sqrt{8\beta c/\lambda}$$

$$= A - \frac{1}{4\sqrt{\beta}}\sqrt{8c/\lambda}$$

Plugging this into (11), one obtains:

$$Z_\lambda = -\frac{1}{\lambda^2} ln\left(\frac{4A\sqrt{\beta}}{\sqrt{8c/\lambda}}\right) + \frac{1}{\lambda}\frac{4\sqrt{\beta}}{\sqrt{8c/\lambda}}\frac{-\frac{c}{\lambda^2}}{4\beta\left(A - \frac{\sqrt{8c/\lambda}}{4\sqrt{\beta}}\right) - 4\beta A}$$

The last term on the right-hand side simplifies to $1/2\lambda^2$, leading to equation (12).

Derivation of the limit used in the proof of Proposition 3

Denote by q_P the derivative of q with respect to P. We compute:

$$lim_{P\to\infty}(A - q)P = lim_{P\to\infty}\frac{(A-q)}{\frac{1}{P}} = lim_{P\to\infty} - \frac{q_P}{-\frac{1}{P^2}} = lim_{P\to\infty}P^2 \cdot \left[\frac{c}{\lambda P^2}\right] = \frac{c}{\lambda}$$

Hence one may take the following limit of term (IV) in equation (15):

$$lim_{P\to\infty}\frac{1}{\lambda}\frac{1}{(A-q)}\frac{c}{\lambda^2 P}=lim_{P\to\infty}\frac{c}{\lambda^3}\frac{1}{P(A-q)}=\frac{c}{\lambda^3}\frac{\lambda}{c}=\frac{1}{\lambda^2}$$