

First result:  $x_2 \geq x_1$

$$\begin{cases} t_1 - c_1(x_1) \geq t_2 - c_1(x_2) \\ t_2 - c_2(x_2) \geq t_1 - c_2(x_1) \end{cases}$$

Adding up:

$$-c_1(x_1) - c_2(x_2) \geq -c_1(x_2) - c_2(x_1)$$

$$c_1(x_2) - c_1(x_1) \geq c_2(x_2) - c_2(x_1)$$

$$\int_{x_1}^{x_2} CMg_1(s)ds \geq \int_{x_1}^{x_2} CMg_2(s)ds$$

$$CMg_1 > CMg_2 \Rightarrow x_2 \geq x_1$$

Second result: first-best is not implementable

First-best:

$$\begin{cases} t_1 - c_1(x_1) = 0 \\ t_2 - c_2(x_2) = 0 \end{cases} \Rightarrow \begin{cases} t_1 = c_1(x_1) \\ t_2 = c_2(x_2) \end{cases}$$

Type 2 may choose contract 1 and obtain:

$$u_2(t_1, x_1) = t_1 - c_2(x_1) > t_1 - c_1(x_1) = 0 = t_2 - c_2(x_2) = u_2(t_2, x_2)$$

That is,  $u_2(t_1, x_1) > u_2(t_2, x_2)$ : type 2 deviates from the first best.

Third result:  $IR_1 \Rightarrow IR_2$  is slack

$$t_2 - c_2(x_2) \geq t_1 - c_2(x_1) > t_1 - c_1(x_1) \geq 0$$

$\checkmark$   
 $IR_2$

Hence  $t_2 - c_2(x_2) > 0$ .

Fourth result:  $IR_1$  is binding

Otherwise, Principal might reduce  $t_1$  and  $t_2$  uniformly so that  $IC'$ s are unaffected,  $IR_2$  would remain active due to the previous result,  $IR_1$  would remain active, and hence all restrictions would be respected with a higher profit for the principal: contradiction.

Fifth result:  $IC_2$  is binding

Assume otherwise. Type 2 still chooses  $x_2$  and his IR is not violated if change is small enough since it wasn't binding.

1 chooses  $x_1$  even more strongly and his IR is unaffected.

Sixth result:  $IC_2$  binding  $\Rightarrow IC_1$  slack.

$$t_2 - c_2(x_2) \stackrel{IC_2 \text{ binding}}{=} t_1 - c_2(x_1) \Rightarrow$$

$$t_2 - t_1 = c_2(x_2) - c_2(x_1) = \int_{x_1}^{x_2} CMg_2(s) ds < \int_{x_1}^{x_2} CMg_1(s) ds = c_1(x_2) - c_1(x_1)$$

Hence:

$$t_2 - t_1 < c_1(x_2) - c_1(x_1)$$

$$t_2 - c_1(x_2) < t_1 - c_1(x_1)$$

Therefore,  $IC_1$  is slack.

7<sup>th</sup> result:  $t_2 \geq t_1$

$$t_2 - c_2(x_2) \geq t_1 - c_2(x_1)$$

$$t_2 - t_1 \geq c_2(x_2) - c_2(x_1) \geq 0$$

Additionally,  $t_2 > t_1$  if  $x_2 > x_1$ .